MATHEMATICS AND SOCIETY—A HISTORICAL VIEW

by Felix E. Browder

Discussions of the history of mathematics tend to fall into one of several distinct patterns. One of these patterns (of which George Mackey's essay on the history of group representations is a particularly distinguished specimen) is to present a critical retrospective essay on the conceptual development of a significant area of mathematical research, illuminating the high points as an exposition of the central ideas of that area in its contemporary state (with some view toward its future prospects). This is the most common pattern for mathematicians active in research who look back on the broad landscape of ideas of which technical details are a part. Its value particularly to mathematical education on a high level is obvious. A less common pattern (and one developed elsewhere in this issue in a typically provocative way by Salomon Bochner) is a historical view of the interaction of mathematical ideas with ideas in a broader cultural or philosophical context. Such analyses are produced (in the infrequent circumstance that they appear at all) by mathematicians with philosophical and historical interests of an unusually broad kind. Their value lies in the contribution they make from this special perspective to the general area of the history of ideas.

The angle of attack of professional historians of mathematics in its most distinctive professional form is exemplified with unusual sharpness and distinction by the essay of Thomas Hawkins on the correspondence of Dedekind and Frobenius on the stages of development of the theory of group characters. Here are the concrete events of mathematical history certified in an unusually authentic form by the existence of direct evidence of what actually happened, which in the most concrete perspective is the central concern of the historian.

My concern here is to take up a theme which is much less obvious as a concern of either mathematician or historian, the historical analysis of the role of mathematics in the various societies in which it has existed or flourished. Such an analysis needs a more explicit justification than the other patterns described above, since it is clearly less spontaneously derived from the natural concerns of the research mathematician or the research historian. In my initial presentation of this theme to the Rice Conference on the History of Analysis, I began without reference to known models or a concretely formed circle of discussion of such issues. Since that time, there has appeared a very clearly formulated and scholarly essay in the same direction by Bos and Mertens [*2*], which has emphasized the significance of this direction of analysis from the point of view of the professional historians of mathematics. My task is therefore reduced to justifying a historical analysis of the social role of mathematics from the point of view of mathematicians themselves.

Aside from the intrinsic interest of the subject (which depends very specifically upon the taste of the individual mathematician), there is one forceful justification for an analysis of the historical vicissitudes of mathematics as a part of society: In the past few years, uncertainty and doubt of the future of mathematics have replaced the certainties and complacency of past decades. Discussions of this future are often based upon assumptions of the *usual* or the *normal*, which have relatively little relation to historical experience of any kind. This is not to say that any kind of historical analysis will give us a clear and reliable guide either to present action or to the future of mathematics. It may provide useful hints or suggestions; the most we can say is that (to provide a phrase suggested by Whitehead's celebrated fallacy of *misplaced concreteness*) it may tend to free us from the most misleading fallacy in thinking about the present and future of mathematics, the assumption of the *eternality of the ephemeral*.

Such a historical analysis of the interaction of mathematics and society is made difficult by a central problem, which appears in a very clear form in Bos and Mertens's essay. The problem is, of course, the controversial and problematic state of the principles of large-scale or historical analysis of social phenomena. Whether one adopts a neo-Weberian point of view (as in the recent book by Joseph Ben-David [*1*] on the social history of science as a whole—probably the most authoritative recent work in the field) or alloys it with neo-Marxist elements (as Bos and Mertens have done to some extent in their summary of a possible treatment of mathematics), or adopts the perspective of some branch of cultural anthropology (as Raymond Wilder has done in his writings—[*18*], for example), there is always a great temptation involved. One is tempted to see mathematics and science as a special case which exemplifies the operation of the general categories of some system of social analysis. I believe that one achieves a good deal more clarity by resisting the temptation, in particular because the categories of these social theories have become clichés in a way that often forecloses the recognition of their very drastic limitations.

The present discussion may be viewed as an extension of some earlier essays [*3* and *4*] on the general role of mathematics, in which the questions we propose to discuss here were raised, but without detailed discussion. Let me begin by putting forward once more a scheme of classifying levels of mathematical activity formulated in [*3*] (which has both similarities to and differences with a corresponding scheme of classification given by Bos and Mertens in [*2*]). The four levels of existence of mathematics as an activity which I propose to distinguish are:

(1) Mathematics as an element of general social practice;
(2) Mathematics as a tool in organized intellectual disciplines;
(3) Mathematics as an autonomous focus of organized and self-conscious intellectual activity (what is generally called mathematical research);
(4) Mathematics as an ideal or transcendent goal of human knowledge or practice.

Across this scheme of classification, there falls a fifth category (put by Bos and Mertens in a parallel status to the preceding four):

(5) Mathematics as a subject of teaching.

This fifth category is a distinctive part of mathematics on all four of the preceding levels, not just a special case of the institution of teaching.

If we examine the first category, mathematics as an element of general social practice, another kind of subclassification becomes useful:

(1a) Mathematics as an element of social organization and control;
(1b) Mathematics as an element of technology and the related analysis of the physical world.

The distinction between these two aspects of mathematical social practice is very far from being a dichotomy (and it is one of the distinct disadvantages of such conventional Marxist categories as *base* and *superstructure* to suggest that it is).

If we examine a broad range of historical evidence, we may begin with the following schematic generalization: Insofar as we can gather from historical evidence (and certainly this is true for all the great historical civilizations), stable and complex social life involved from an early stage the social application of the mathematical activities of counting and measuring. Such application developed in a succession of forms:

(I) The organization of social time through the calendar.
(II) The organization of social exchange, eventually through the introduction of coins and other monetary units.
(III) The measurement of units of land use and ownership.
(IV) The introduction of formal institutions for teaching writing and mathematics (usually as part of the sacred institutions of the great civilizations).
(V) The use of mathematics as an organizing instrument of relatively

> sophisticated forms of military activity, as a way of organizing the action of a relatively unwieldy mass of soldiers under conscious and centralized control.

The great historical civilizations whose remains have been intensively studied—Babylon, Egypt, China, India, and the Meso-American civilizations—all possessed a well-developed calendar at very early stages of the historical record. In the case of the Maya civilization, a complex system of time-keeping flourished while the wheel was never developed. Some writers [*10*] have even suggested that the calendar was the central feature of cultural development in Neolithic and late Paleolithic times. Others [*5, 6, 9, 14, 16*] have boldly speculated on the existence of sophisticated, preliterate, mathematical and astronomical cultural complexes.

Whatever the truth may be concerning such historical speculations, the incontrovertible historical evidence for the intensely developed mathematical activity of the Babylonians as discovered and analyzed by Neugebauer and his successors [*13*] is overwhelming. Less imposing but substantial evidence exists for a reasonable standard of mathematical practice in Egypt, China, classical India, and particularly in the Maya civilization. In many cases, mathematical activity is distinctively associated with a special caste of priests and temple scribes. Like other cultural features of these hierarchical and traditional societies, such as their art, their literature, and their social organization, the mathematical practice of the great castes of temple scribes emphasized traditional skills and dogma. There was little room or incentive for criticism or for outbursts of creative innovation.

When mathematics arose as an autonomous and self-conscious intellectual discipline in Greece in the seventh and sixth centuries B.C., it was in a social context almost diametrically opposed to the stability and conservatism of the Babylonian, Egyptian, and Minoan civilizations. In the Ionian fringe of Asia Minor, and shortly thereafter across much of the Greek world, there was intense social flux, conflict, and instability. An equally intense and self-conscious intellectual life developed with remarkable speed, of which mathematics was a conspicuous part. On the one hand, it is tempting to speculate [*7* and *8*] that the very intensity of the criticism and controversy, the *agonistic* element, so characteristic of Greek intellectual, social, and political life in this period, was the matrix in which the concepts of precise logical reasoning from indubitable premises (the unique Greek contribution to the world-view of mathematics and of the intellect) would naturally arise. After all, plausibility arguments are useful mainly for convincing those who are friendly and well-disposed, rigorous proofs for those who are critical and hostile on principle. Certainly, the use of rigor in deduction, and particularly use of the newly introduced practice of reasoning to absurdity in philosophical argument, vindicates this generalization.

On the other hand, for those influential thinkers who, like Plato and the Pythagoreans, placed a strong emphasis on social and intellectual stability, mathematics and its close relations, astronomy and music, were of decisive importance precisely as intellectual disciplines capable of systematic rational and orderly development. The emphasis upon the moral and intellectual value of mathematics in education, as formulated by Plato and his disciples, survived as an active principle of Greek thought about education long after the destruction of the society in which it originated. It eventually became the foundation of the seven liberal arts transmitted to Western Europe from the classical world during the Dark Ages, from which the framework of education and learning was reconstructed.

After two or three centuries, the most intensely creative period of classical Greece in all areas of intellectual activity was ended with the conquests of Alexander, the creation of the Hellenistic kingdoms, and the exportation of the Greek intellectual tradition to the great Hellenistic cities of Alexandria, Antioch, and Pergamum, as well as to the remaining centers of Greece and Magna Graecia in southern Italy and Sicily. This was the age of Euclid, Archimedes, and Apollonius, and the final consummation of Greek geometry and logic. It was also the era in which the scattered activities of Greek mathematicians, who were given a distinct identity and self-consciousness in the classical period by such dedicated spokesmen as the Pythagoreans and Plato but who lacked stable institutions (aside from private academies like Plato's), were put on a new and different social basis. The paradigm was the great library of Alexandria, the first institution of intellectual research and education founded upon and supported by a state on a secular basis. It was the central nexus of a widespread network of intellectual institutions across the Hellenistic world, which maintained their intellectual supremacy (if not their intellectual vigor) as long as the Mediterranean world survived.

The central feature of the extraordinary development of mathematics among the Greeks (and for that matter of the whole of Greek intellectual life) was the creation of the concepts of *truth* and of *knowledge* in all their shadings and ramifications. It has been argued by Joseph Needham [*12, 11*] that it is the existence of this mathematically based tradition stemming from the Greeks that has made possible the scientific creativity of modern Western Europe as compared to China.

Mathematics was created in a sophisticated form by the Greeks, as an explicit and autonomous form of knowledge and as an ideal of human knowledge. Though mathematics was essential and increasingly used in Greek astronomy, particularly after its interaction with late Babylonian observational astronomy, neither this use nor the relatively restricted practical applications of mathematics in technology (by Heron of Alexandria and others) seems to have been of any serious consequence to the basic practice of Greek mathematics (see [*15*]). Though the Romans who conquered the Hellenistic world had no taste and aptitude for Greek intellectual achievements in mathematics and the sciences, no separate traditions of a rational order arose to challenge the Greek conception. The Romans might use Heron's books as manuals for building and military purposes, but they did not advance them one step. Nor, after a while, did the Greek inheritors of the classical and Hellenistic traditions do much more. By the time of the barbarian invasions and conquests, the creativity of the mathematical and scientific thought of the Greek world had decayed.

Yet the death of the active Greek mathematical tradition left a decisive legacy for the future development of mathematics and of science, embodied in several forms. Its most direct influence was the transmission of the educational tradition of the Greeks, particularly in its neo-Platonic form, through such writers as Boethius and Cassiodorus. Of more importance in the long run was the preservation, particularly by the Arabs, of many of the great Greek mathematical works, particularly those of Euclid, Apollonius, and Archimedes. Moreover, in the Islamic world, a circle of mathematical writers tended to bring together the logical and geometric traditions of the Greeks with the rather different techniques of calculation and algebraic manipulation embodied in the Babylonian and Indian traditions.

The rebirth of mathematical creativity that occurred in Western Europe during the Renaissance and the following centuries took place in a context (particularly in Italy) which had both striking similarities to and differences from the social context of classical Greece. There was a common element of political struggles, individual and intellectual self-assertiveness, and social flux. On the other hand, the sharp growth of adventuresome intellectual innovation was sharply restricted by the power of the Roman Church, particularly after the Council of Trent. The mathematicians of the epoch were isolated and contentious individuals without accepted social status or function. A few were professors in the recently founded universities of the major Italian cities, others were free-lance adventurers with a dozen occupations like Cardano, and the most accomplished, such as Galileo, survived on sporadic patronage from princely families and oligarchic city governments.

In this period the decisive emphasis had shifted (despite the awe evoked by the Greeks) from the self-contained and purely logical universe of Greek geometry to the use of mathematics as a universal tool for the total understanding of the world and especially the mathematized physical cosmos. Whether this was an "internal" development stemming from the absorption of the algebraic innovations of the Arab tradition, or an "external" development based upon the commercial arithmetic of the Italian cities of the Renaissance and the rapidly developing European economies, the practical emphasis of the new tradition of the "analytic" in mathematics was unmistakable. It merged very soon with the ideologically and religiously controversial development of Copernican astronomy and Galilean physics.

When mathematical activity moved to France and the Low Countries, it was practiced and carried forward by jurists like Viète and Fermat and soldier-adventurers like Descartes on the one hand, and professional scholars like Huygens on the other. The same diversity of social status can be seen in the university professors Barrow and Newton in England (with Barrow to become chaplain to the King), and the cosmopolitan courtier Leibniz. The few European mathematicians in the seventeenth and eighteenth centuries were supported principally by newly founded scientific academies like those of Paris, Berlin, and St. Petersburg. The latter two gave the incomparably fertile Euler the opportunity to practice an uninterrupted career as a creative mathematician (according to the astounding estimate of Clifford Truesdell [*17*], Euler was to produce forty percent of the mathematical papers published between 1750 and 1850 in the Proceedings of both the Berlin and St. Petersburg Academies).

The impact of a mathematical physical science during the seventeenth and eighteenth centuries has been labelled a "Scientific Revolution" by historians because of its dramatic effect on the thinking of the educated classes of Western Europe and thereby on the whole of their societies. Yet it was not accompanied by any massive change of status for its practitioners, except for the handful of members of the great academies. Whatever the background influence of broad groups of "philomaths," mathematical amateurs, and "mathematical practitioners" who have been identified and studied by historians, the creation of stable social forms for mathematical activity had to wait for the nineteenth century. The same conclusion seems to hold even for the development of mathematical education. While the educational reforms of the French Revolution and of Napoleon gave rise to the Ecole Polytechnique in Paris and systematic forms of mathematical teaching on a high level, this innovation was not soon imitated elsewhere. According to Felix Klein, the first systematic teaching on anything approaching a research level in mathematics in German universities was begun by Jacobi in Köningsberg in 1841.

The nineteenth century saw the dramatic transformation of the intellectual disciplines into professions as a corollary of two major movements: the industrial revolution and the transformation of the universities in Western Europe. The industrial revolution for the first time vindicated Francis Bacon's dream of the practical power of science. Although the intellectual achievements of science, particularly in the eighteenth-century Enlightenment, may have created a receptive climate for the assertion of a secular practical bourgeois attitude toward the world, the technological effects of science itself were not considerable until the nineteenth-century industrial revolution. Late in the century the first scientifically based industries, first in the chemical field, and then in the electrical, dramatically changed the social role of the scientific disciplines.

An earlier and more drastic influence in the evolution of the mathematics profession was the creation of the modern university system, begun by W. von Humboldt in revising the charter of the University of Berlin in 1809, and followed throughout Germany and then the rest of Europe over the course of the century. The introduction of professorships in mathematics and natural sciences, and the emerging role of the university professor as an active participant and organizer of research led to a systematization and broadening of the whole process of mathematical research and innovation. Within a few decades after the system took hold, its participants began to believe that it had always been and would last forever without effort or thought.

The ingestion of the industrial revolution into the university also took place first in Germany, under the dedicated leadership of Felix Klein. Through his political and intellectual influence in the German official and academic world, applied science and applied mathematics were introduced into Göttingen around the beginning of the twentieth century. Applied mathematics, in the sense of mathematics specifically directed to industrial and military concerns, grew steadily in the West from this beginning through the dramatic social and political transformations initiated and accelerated by the two world wars.

The Second World War especially was the transition point for a remarkable transformation of the social structure of science and education, which has had significant effects upon the social interactions of mathematics. The decades that followed the war saw the rapid expansion of all the scientific professions, including the mathematical, through the massive expansion of government financing and the resulting growth of the higher education system. The basic educational expectation of a large mass of the youth was raised from high school graduation to college graduation. Vast new industries based upon science and mathematics were created, including electronics and the computer industry.

After the initial explosive expansion, however, the ebb and flow of government financing caused massive dislocations in the scientific professions. These were mitigated for mathematics only by its involvement in its historically most stable function, basic mathematical education up to the level prescribed by a growing need for mathematical skills in social practice and in various professional disciplines. At the same time, mathematically-based techniques have been applied through such increasingly fashionable areas as systems theory to social control by centralized decision-making—another historical instance of both the possibilities and dangers of mathematics as a tool of social organization, particularly when it is allied to computer technology.

It is tempting to complete this historical panorama by drawing some morals, which at the outset was implicitly promised by our suggested justification for the enterprise. Let me cite two very simple ones:

(a) Nothing is permanent, not even what we most treasure, and not even relatively stable without great effort and thought.

(b) Despite the collapse of civilizations, the mathematical tradition has survived about 2700 years since the ancient Greeks. If we care enough about its intellectual values, it will survive a good while longer.

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